October 1975 B75-10273

NASA TECH BRIEF

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A Study of Accuracy in Selected Numerical-Analysis Integration Techniques

A report, "Some Self-Starting Integrators," has been published discussing several methods of performing numerical integration with a computer. The increased use of sequential data processors (e.g., Kalman filters) to handle large quantities of data requires software systems that can make accurate predictions of future data based on current data.

When the data can be expressed as a state vector $\underline{x}$ that is the dependent variable in a differential equation, self-starting integrators can be used to predict future data. An example is predicting the position of a moving vehicle. From a position/velocity measurement, a Kalman filter and the integrators can be used to help predict future positions, compare them with actual positions, and apply course corrections.

A self-starting integrator is one which takes a current estimate of a state vector ($\underline{x}_i$) (no previous-position vectors are needed) and, using the equation $\dot{\underline{x}} = \underline{f}(\underline{x},t)$, propagates the estimate forward a time Δt. Taylor-series integrators and 2d and 4th order Runge-Kutta integrators are numerical-analysis techniques that have been generally used for this purpose. For many applications the best particular technique and its degree of accuracy have not been well known.

The report makes comparisons between Taylor-series integrators and various types and orders of Runge-Kutta integrators. A numerical method of determining integrator accuracy over each step is shown. Taylor-series and 2d, 4th, and 5th order Runge-Kutta integrators are discussed. No exact solution to the 5th order Runge-Kutta is known. It is shown that with vector differential equations six derivatives must be evaluated for the 5th order integrator rather than the five that might be expected.

Two sets of approximate 5th order integrator constants are shown and evaluated. When compared to 4th order Runge-Kutta integrators, they give only slightly more accurate solutions. For this reason it is felt that the 4th order method is more suitable for most applications.

Finally, a new set of equations is presented for integrating onboard accelerometer data. These accelerometers usually read out the integral of the sensed acceleration over a given period of time rather than the instantaneous acceleration. A set of equations is reported for integrating the accelerometer data more accurately than the series of short-step integrations formerly used.

Notes:

1. This study is described in the following report:
   Some Self-Starting Integrators for $\dot{\underline{x}} = \underline{f}(\underline{x},t)$
   NASA CR-141784 (N75-24419).

   Copies of this report may be obtained at cost from:
   Technology Application Center
   University of New Mexico
   Albuquerque, New Mexico 87131
   Telephone: (505) 277-3622
   Reference: B75-10273
2. Specific technical questions may be directed to:
   Technology Utilization Officer
   Johnson Space Center
   Code AT3
   Houston, Texas 77058
   Reference: B75-10273

Patent status:

NASA has decided not to apply for a patent.

Source: W. M. Lear of
TRW, Inc.
under contract to
Johnson Space Center
(MSC-14802)

Category: 09 (Mathematics and Information Sciences)